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COMPARISON OF BAROTROPIC AND BAROCLINIC NUMERICAL FORECASTS AND CONTRIBUTIONS OF VARIOUS EFFECTS

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ABSTRACT

500-mb. barotropic and 500-mb. baroclinic numerical forecasts for two cases, a developing baroclinic cyclone and a quasi-barotropic cyclone, are presented and compared. The barotropic forecasts did not indicate accurately the changes in circulation or the magnitude of the height falls ahead of the circulation maxima. 700-mb. forecasts from the same initial times as the 500-mb. barotropic and baroclinic forecasts, for each of the four terms of the frictionless vorticity equation, are presented. These 700-mb. forecasts are compared with each other and in added combinations with the 700-mb. verifications and with 700-mb. barotropic forecasts. These comparisons are then used diagnostically in an analysis of the errors in the 500-mb. barotropic forecasts. Each of the four terms of the vorticity equation is discussed. An explanation for the success of the barotropic forecasting model is suggested. Contributions of the horizontal velocity divergence, vertical advection of vorticity, and twisting terms to errors in the barotropic forecasting model are discussed in some detail. It is concluded that the major problem in developing a successful baroclinic forecasting model to substitute for the existing barotropic forecasting model is that of determining in space and time an accurate approximation of the vertical profile of vertical motion.

1. INTRODUCTION

A technique frequently applied in our efforts to improve weather prognosis is that of making a detailed case study of a weather situation which allows isolation of the problem of the moment. The problem under study is baroclinic development. The purpose of this paper is to present, first, the results of a study and comparison of barotropic and baroclinic numerical forecasts from the same initial time for a case of baroclinic development. As a check on the results of the first case, a similar study was made for a case presumed to involve little or no baroclinic development—a quasi-barotropic case; these results are presented also. The analyses and 500-mb. forecasts used in the two studies were selected from the Joint Numerical Weather Prediction (JNWP) Unit and National Weather Analysis Center (NAWAC) operational files. The 700-mb. forecasts were prepared specially in JNWPU for these two studies. 500-mb. barotropic forecasts, S_2 Model[2] [1] and 500-mb. baroclinic forecasts, Thermotropic Model[3] [2] were compared.

The figures shown speak for themselves. The conclusions arrived at are the most obvious. In general they point up some inadequacies of the barotropic model and strongly suggest the inclusion of the horizontal velocity divergence, vertical advection of vorticity, and twisting terms in numerical weather prediction models. It is recalled that in the recent past the capacity of the electronic computers available for numerical weather prediction limited greatly the forecasting model. As computers increase

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[2] The S_2 Model employs a non-divergent wind which is approximately geostrophic.
[3] The Thermotropic Model employs the geostrophic wind.

in capacity and speed, it is anticipated that the more successful forecast models developed and subsequently employed will include most of the baroclinic terms and will produce greatly improved numerical forecasts.

2. APPROACH TO THE PROBLEM

It was decided that in comparing 500-mb. barotropic and 500-mb. baroclinic forecasts it would be necessary to know more about the detailed behavior of the atmosphere than would be immediately apparent from attempting to resolve differences in basic forecasting equations, forecast contour fields, forecast height error fields, etc. Therefore, additionally, 700-mb. 12-hour height tendency forecasts were made from data for the same initial times as the 500-mb. forecasts. The techniques employed are comparable to those employed by Arnason [3] and Winston [4] except computations of vertical motion were not necessary since fields of large-scale 500-mb. vertical motion for the initial times (figs. 1F and 5F) were already computed and available as products of the thermotropic forecast [2,5].

The frictionless vorticity equation—

$$\frac{\partial \zeta}{\partial t}=-\mathbf{V}\cdot\nabla\eta+\eta\,\frac{\partial\omega}{\partial p}-\omega\,\frac{\partial\eta}{\partial p}-\left(\nabla\omega\times\frac{\partial\mathbf{V}}{\partial p}\right)\cdot\mathbf{k}, \qquad (1)$$

in which t is time; ζ is relative vorticity; p is pressure; $\mathbf{V}$ is the horizontal wind vector; η is absolute vorticity; ω is the individual change of pressure with time, $\frac{dp}{dt}$, [6, 7]; and $\mathbf{k}$ is the unit vertical vector—was separated into four finite difference equations from which each right-hand-side term could be evaluated for 700 mb. and its field of values relaxed to obtain a 12-hour, one-time-step height tendency forecast for 700 mb. The terms on the right-hand side of equation (1) are, reading from left to right, horizontal advection of absolute vorticity, horizontal velocity divergence, vertical advection of vorticity, and twisting of the vortex tubes; these will be referred to as the horizontal advection term, the divergence term, the vertical advection term, and the twisting term, respectively.

First let

$$\frac{\partial\zeta}{\partial t}=\left(\frac{\partial\zeta}{\partial t}\right)_1+\left(\frac{\partial\zeta}{\partial t}\right)_2+\left(\frac{\partial\zeta}{\partial t}\right)_3+\left(\frac{\partial\zeta}{\partial t}\right)_4 \qquad (2)$$

in which each right-hand-side term to be evaluated at 700 mb. represents the corresponding term in equation (1). Then an equation, in which $\Delta t=12$ hours (and therefore $s=43{,}200$, the number of seconds in 12 hours), for the horizontal advection term can be written,

$$\left(\frac{\partial\zeta}{\partial t}\right)_1=-s(\mathbf{V}\cdot\nabla\eta) \qquad (3)$$

in which

$$\left(\frac{\partial\zeta}{\partial t}\right)_1=\frac{g}{f}\left(\nabla^2\,\frac{\partial z}{\partial t}\right)_1=\frac{gm^2}{fd^2}\left[\left(\frac{\Delta z}{\Delta t}\right)_n+\left(\frac{\Delta z}{\Delta t}\right)_e+\left(\frac{\Delta z}{\Delta t}\right)_s+\left(\frac{\Delta z}{\Delta t}\right)_w-4\left(\frac{\Delta z}{\Delta t}\right)_0\right]_1$$

and

$$-s(\mathbf{V}\cdot\nabla\eta)=\frac{gs}{f}\,\mathbf{J}(\eta,z)=\frac{gsm^2}{4fd^2}\,[(\eta_e-\eta_w)(z_n-z_s)-(\eta_n-\eta_s)(z_e-z_w)].$$

For evaluation at 700 mb. this reduces to

$$\left[\left(\frac{\Delta z}{\Delta t}\right)_n+\left(\frac{\Delta z}{\Delta t}\right)_e+\left(\frac{\Delta z}{\Delta t}\right)_s+\left(\frac{\Delta z}{\Delta t}\right)_w-4\left(\frac{\Delta z}{\Delta t}\right)_o\right]_1 -10.8\,[(\eta_e-\eta_w)(z_n-z_s)-(\eta_n-\eta_s)(z_e-z_w)]_{700}=R_1\approx 0.$$

Likewise an equation for the divergence term can be written,

$$\left(\frac{\partial\zeta}{\partial t}\right)_2=s\eta\,\frac{\partial\omega}{\partial p} \qquad (4)$$

and reduced to

$$\left[\left(\frac{\Delta z}{\Delta t}\right)_n+\left(\frac{\Delta z}{\Delta t}\right)_e+\left(\frac{\Delta z}{\Delta t}\right)_s+\left(\frac{\Delta z}{\Delta t}\right)_w-4\left(\frac{\Delta z}{\Delta t}\right)_o\right]_2 -1.204\,\sin\psi(1+\sin\psi)^2\eta_{700}W_{500}=R_2\approx 0.$$

An equation for the vertical transport term can be written

$$\left(\frac{\partial\zeta}{\partial t}\right)_3=-s\omega\,\frac{\partial\eta}{\partial p} \qquad (5)$$

and reduced to

$$\left[\left(\frac{\Delta z}{\Delta t}\right)_n+\left(\frac{\Delta z}{\Delta t}\right)_e+\left(\frac{\Delta z}{\Delta t}\right)_s+\left(\frac{\Delta z}{\Delta t}\right)_w-4\left(\frac{\Delta z}{\Delta t}\right)_o\right]_3 -1.084\,\sin\psi(1+\sin\psi)^2W_{500}(\eta_{1000}-\eta_{500})=R_3\approx 0.$$

An equation for the twisting term can be written,

$$\left(\frac{\partial\zeta}{\partial t}\right)_4=-s\left(\nabla\omega\times\frac{\partial\mathbf{V}}{\partial p}\right)\cdot\mathbf{k} \qquad (6)$$

and reduced to,

$$\left[\left(\frac{\Delta z}{\Delta t}\right)_n+\left(\frac{\Delta z}{\Delta t}\right)_e+\left(\frac{\Delta z}{\Delta t}\right)_s+\left(\frac{\Delta z}{\Delta t}\right)_w-4\left(\frac{\Delta z}{\Delta t}\right)_o\right]_4 -.0133\{[W_e-W_w]_{500}\cdot[(z_e-z_w)_{1000}-(z_e-z_w)_{500}] +[W_n-W_s]_{500}\cdot[(z_n-z_s)_{1000}-(z_n-z_s)_{500}]\}=R_4\approx 0.$$

In these equations $\Delta\,t=12$ hours; $g=9.8$ m.sec.$^{-2}$; f is the Coriolis parameter; ψ is latitude: $m=\frac{1+\sin\,60°}{1+\sin\,\psi}$, the map magnification factor on a polar stereographic projection true at 60° latitude; $d=381$ km., the mesh size; W is vertical velocity in millimeters per second; η is absolute vorticity in units of sec.$^{-1}\times10^{-4}$; z is feet, except dekafeet in the terms z_n, z_e, z_s, and z_w; and R is the residual. Subscripts 1000, 700, and 500 designate the pressure surface at which the value is determined. Subscripts n, e, s, and w designate, in clockwise rotational order on a square mesh grid, values at the four grid points immediately surrounding a central grid point value designated with subscript o. Assumed in these equations are: (1) geostrophic velocity and vorticity at 1000 mb., 700 mb., and 500 mb.; (2) a parabolic profile of vertical velocity, $\frac{dz}{dt}$, between 1000 mb. and 500 mb. with $W=0$ at 1000 mb. and $W_{700}=0.7\ W_{500}$; (3) constant density values at 700 mb. of $\rho=9\cdot10^{-4}$ tons m.$^{-3}$ and at 500 mb. of $\rho=7\cdot10^{-4}$ tons m.$^{-3}$; and $\omega=-\rho g W$.

Over a 14×17 point grid field the boundary of which for each case is the edge of the geographical area shown in figures 1–8, grid point values from the 1000-mb., 700-mb., and 500-mb. pressure surfaces for height, latitude, and vertical velocity were determined from initial analyses and data. The absolute vorticity for each grid point of the three pressure surfaces was computed by entering a graph, specially prepared for the projection and $1:20\cdot10^6$ scale chart used, with the value of the computed finite difference height Laplacian and latitude. Values of the horizontal advection term were not computed on the outside boundary or relaxed on the adjacent inner boundary; grid point values for all four terms for 700 mb. were computed only for the inner 10×13 point grid. The field of grid point values of each individual term was then relaxed by hand using Southwell's method to obtain four 12-hour one-time-step height tendency forecasts for 700 mb. These forecasts were then added graphically in several combinations. Additionally, to obtain an independent estimate of the contribution of the horizontal advection term to a 12-hour height change at 700 mb., 700-mb. baratropic forecasts, S_2 Model [1], were made on the electronic computer.

3. A BAROCLINIC CASE

The considerations in selecting a baroclinic case for study were: (1) a measurable increase in circulation to occur in a young cyclone within a 12-hour period at both the surface (figs. 1A and 1B) and 500 mb. (figs. 1C and 1D); (2) the cyclone to be situated over the relatively flat Plains region of the United States and southern Canada, to insure minimum terrain effects on vertical motion and dense data coverage for accurate analyses; and (3) the cyclone to be associated with a baroclinic atmosphere as evidenced by the out-of-phase relation of 1000-mb. to 500-mb. thickness lines and 500-mb. contours (compare figs. 1C and 1E in the region immediately north of Montana).

The 500-mb. barotropic 12-hour forecast (fig. 2A) did not indicate accurately the increase in circulation that occurred in the developing trough over north central United States (fig. 1D) or the magnitude of height fall just ahead of the circulation maximum (fig. 2 B, C, and D). The 500-mb. thermotropic 12-hour forecast (fig. 2E) indicated errors of the same sign but was definitely superior to the barotropic forecast in the immediate region of the circulation maximum (figs. 2D and 2F). The barotropic model forecasted the heights to be too high over most of the map area shown. Since both 500-mb. forecasts were computed over a 31×34 point grid area (a number of grid lengths larger than the map area shown), boundary effects are considered to be unimportant.

The major difference in the forecasting equations employed by the two models is a baroclinic term of the Thermotropic Model, $-K(\mathbf{V}_T\cdot\nabla\zeta_T)$, comparable to a right-hand term of equation (1) in which K is an empirically determined positive constant and $\mathbf{V}_T$ and ζ_T are thermal velocity and thermal relative vorticity, respectively, for the 1000-mb. to 500-mb. layer. This term appears to be related to the process whereby potential energy is converted to kinetic energy and it can be credited for the difference between the 500-mb. thermotropic forecast and the 500-mb. baratropic forecast. From studying subjectively numerous 500-mb. thermotropic forecasts it appears that this term forecasts continuous baroclinic development. Its cumulative contribution can result in serious contamination, especially in the longer-period forecasts.[4]

The 700-mb. initial analysis (fig. 3A) is similar in appearance to the 500-mb. initial analysis (fig. 1C). The major difference is that the trough and ridge line positions at 700 mb. were east of the 500-mb. positions as is normal for moving systems in westerly flow. The same was true of the trough and ridge lines 12 hours later (figs. 3C and 1D) and also of the observed centers of 12-hour height change (figs. 3D and 2D). But the magnitudes of the observed 12-hour height change centers were considerably less at 700 mb. than at 500 mb. The 700-mb. 12-hour forecast height change (fig. 3B), obtained by graphically adding the forecasts for all four terms (figs. 4C–F), compared favorably with the observed height change (fig. 3D). However, the height fall center associated with the developing cyclone was underforecast in speed of movement and magnitude (−240 feet forecasted as compared to −310 feet observed). These differences can be explained to some degree by the fact that in the atmosphere changes operated to cause additional changes continuously throughout the 12-hour period whereas only the initial conditions were considered in making the 12-hour one-time-step forecast. The 700-mb. barotropic forecast (figs. 3E and 3F) can be compared with the 12-hour tendency forecast for the horizontal advection term (fig. 4C). The major difference between these two forecasts is in the speed of movement of the height fall center. Neither even closely

[4] In the process of converting to a new computer and a larger grid area the Joint Numerical Weather Prediction Unit discontinued routine forecasting with the Thermotropic Model in June 1957.

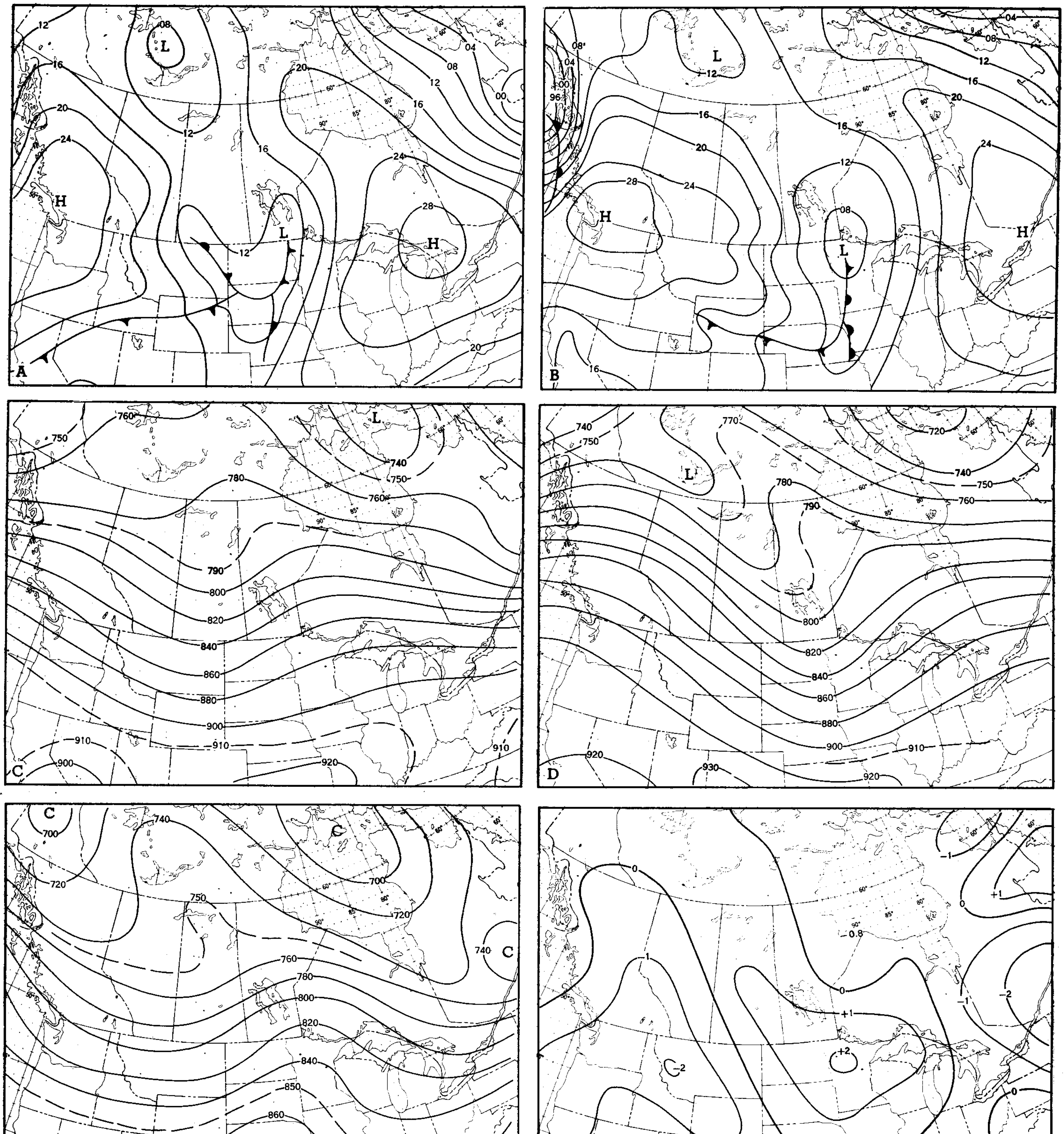

FIGURE 1.—(A) Surface analysis, 1230 GMT, Oct. 5 and (B) 0030 GMT, Oct. 6, 1956. (C) 500-mb. analysis, 1500 GMT, Oct. 5, and (D) 0300 GMT, Oct. 6, 1956. (E) 1000 to 500-mb. thickness, 1500 GMT, Oct. 5, 1956. (F) 500-mb. vertical velocity in cm.sec.$^{-1}$, 1500 GMT, Oct. 5, 1956.

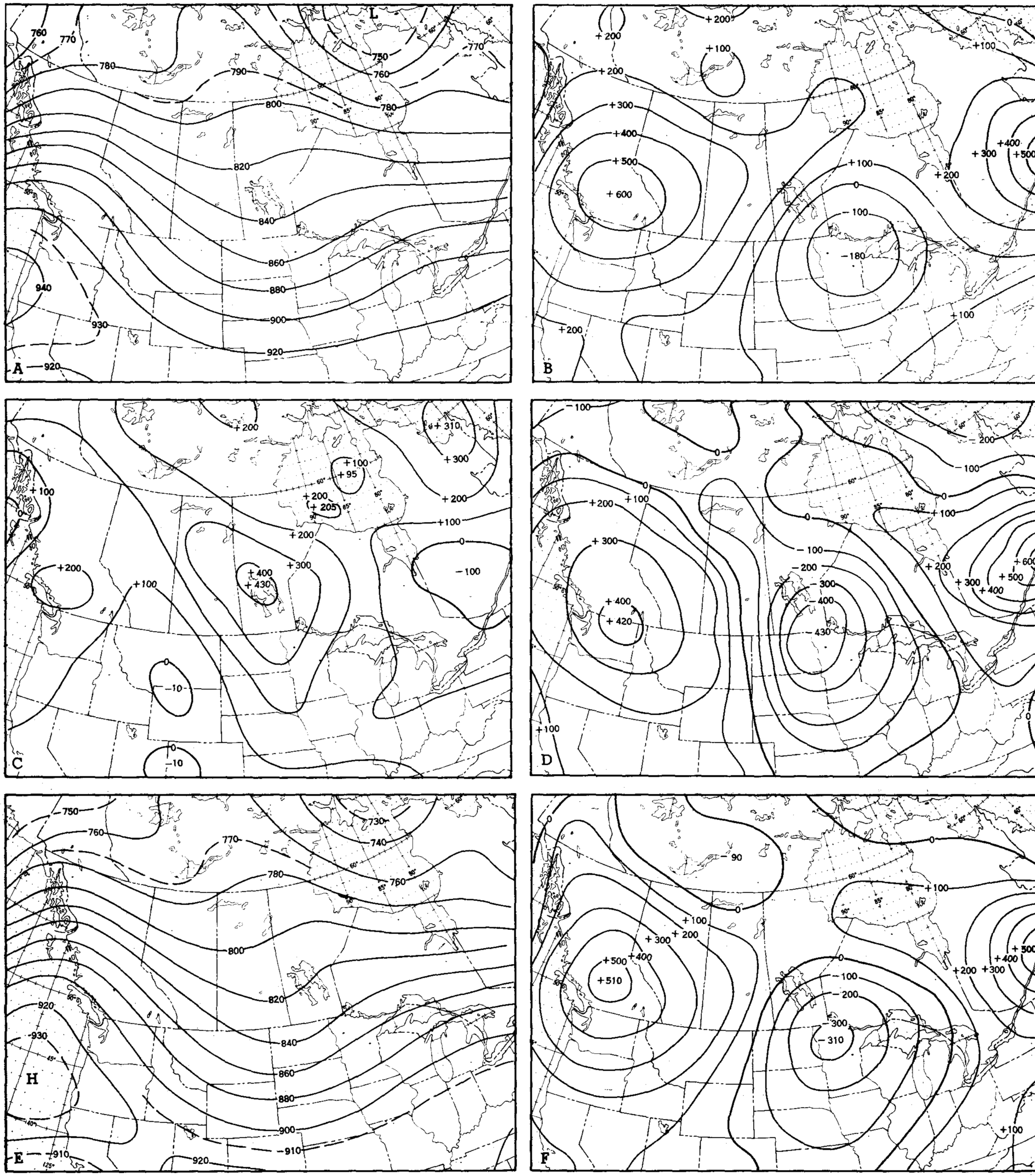

FIGURE 2.—(A) 500-mb. barotropic 12-hour forecast from 1500 GMT, Oct. 5, 1956 and (B) the height change (in feet) it represents. (C) Error of forecast height change. (D) Observed 12-hour height change from 1500 GMT. (E) 500-mb. thermotropic 12-hour forecast from 1500 GMT and (F) the height change it represents.

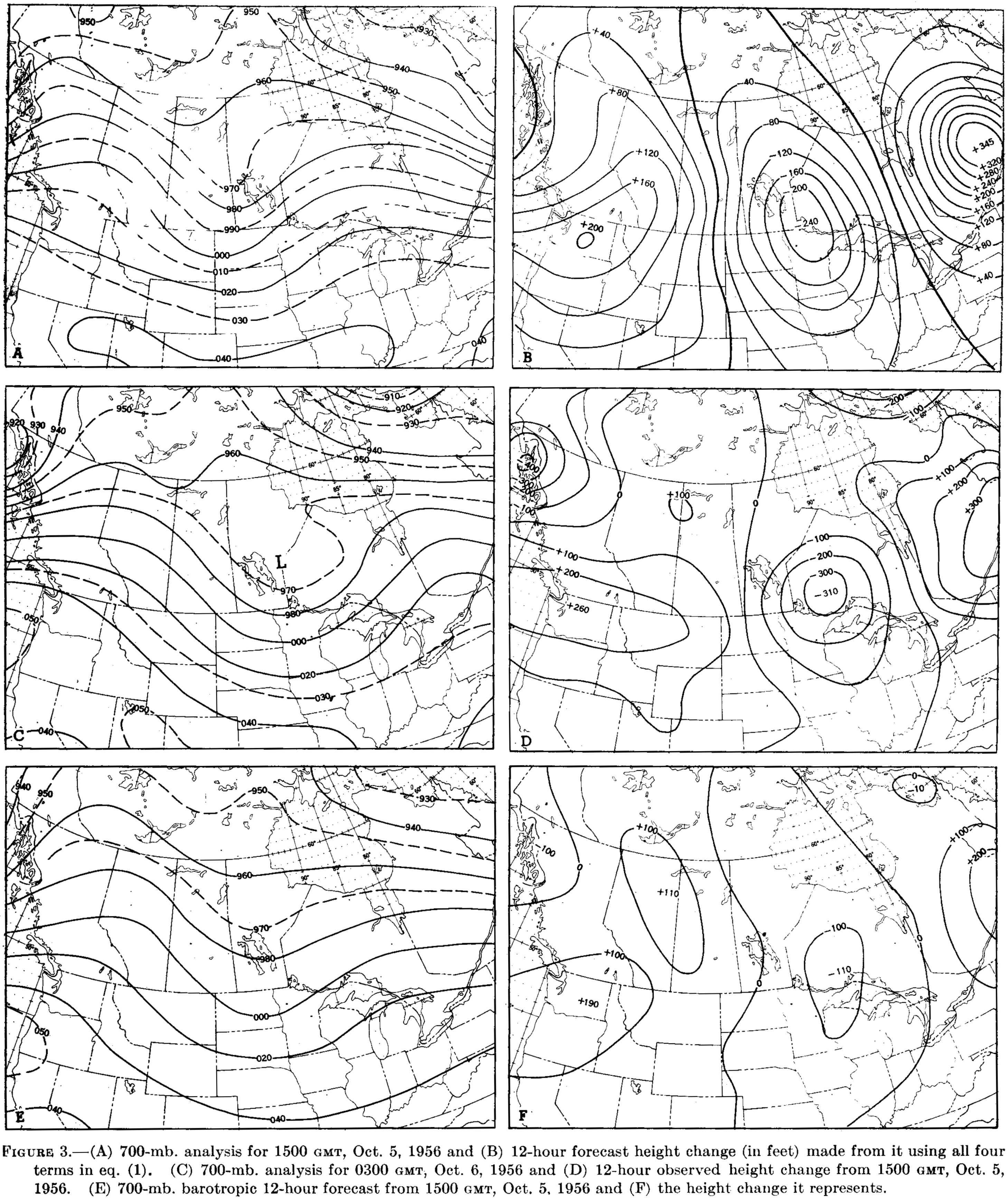

FIGURE 3.—(A) 700-mb. analysis for 1500 GMT, Oct. 5, 1956 and (B) 12-hour forecast height change (in feet) made from it using all four terms in eq. (1). (C) 700-mb. analysis for 0300 GMT, Oct. 6, 1956 and (D) 12-hour observed height change from 1500 GMT, Oct. 5, 1956. (E) 700-mb. barotropic 12-hour forecast from 1500 GMT, Oct. 5, 1956 and (F) the height change it represents.

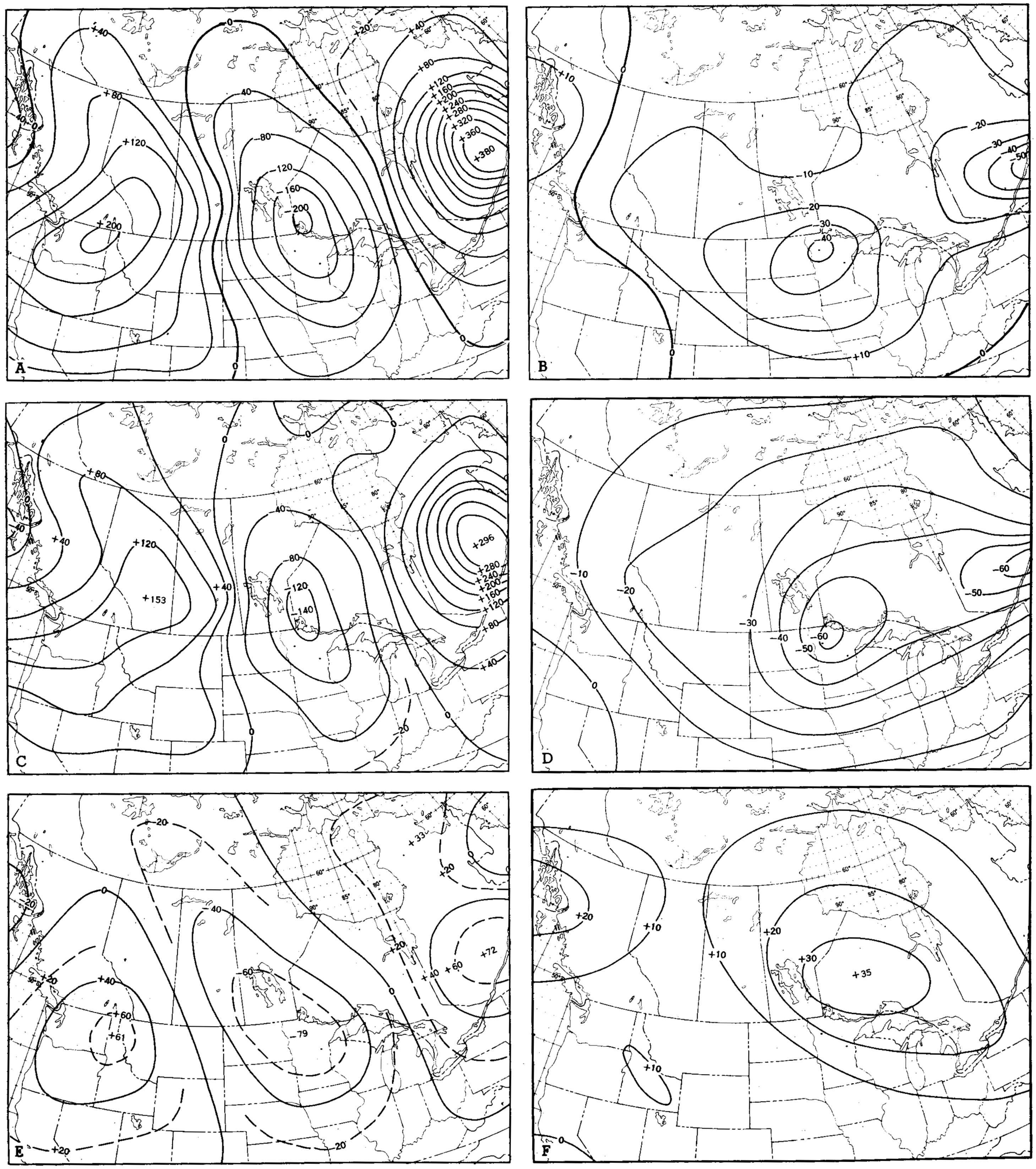

FIGURE 4.—700-mb. 12-hour forecast height change (in feet) from 1500 GMT, Oct. 5, 1956 made from (A) horizontal advection and divergence terms, $-\mathbf{V}\cdot\nabla\eta+\eta(\partial\omega/\partial p)$; (B) vertical advection and twisting terms, $-\omega(\partial\eta/\partial p)-[\nabla\omega\times(\partial\mathbf{V}/\partial p)]\cdot\mathbf{k}$; (C) horizontal advection term, $-\mathbf{V}\cdot\nabla\eta$; (D) vertical advection term, $-\omega(\partial\eta/\partial p)$; (E) divergence term, $\eta(\partial\omega/\partial p)$; (F) twisting term, $-[\nabla\omega\times(\partial\mathbf{V}/\partial p)]\cdot\mathbf{k}$.

approximated the magnitude of the observed 700-mb. height fall (fig. 3D). The 700-mb. forecast obtained by combining all four terms (fig. 3B) was definitely superior to all other 700-mb. forecasts.

4. A QUASI-BAROTROPIC CASE

The considerations in selecting a barotropic case were: (1) no increase in circulation to occur in an old, unchanging cyclone for a 12-hour period at both the surface (figs. 5A and 5B) and 500 mb. (figs. 5C and 5D); (2) the cyclone to be situated over the Plains region for the reasons stated in selecting a baroclinic case; and (3) a minimum of baroclinicity to be in evidence (compare figs. 5C and 5E). Actually an increase in circulation at 500 mb. of approximately 40 percent of the initial absolute vorticity occurred in 12 hours. This can be explained as a baroclinic effect which is evident in the initial out-of-phase orientation of 1000-mb. to 500-mb. thickness lines and 500-mb. contours to the south of the cyclone center. But during the same 12-hour period no appreciable increase in circulation occurred at the surface or at 700 mb. (figs 7A and 7C). Twelve hours after 1500 GMT, December 24, 1956, the cyclone at 500 mb. was no longer identifiable as a closed center of circulation but had been instrumental in intensifying its associated eastward-moving major trough in the westerlies.

As in the baroclinic case, the 500-mb. barotropic 12-hour forecast (fig. 6A) did not indicate either the circulation that occurred (fig. 5D) or the magnitude of the height fall (figs. 6B and 6D) ahead of the cyclone. Again the 500-mb. thermotropic 12-hour forecast (fig. 6E) was superior to the 500-mb. barotropic forecast but in this case it indicated too great a magnitude of height fall (fig. 6F) ahead of the cyclone. The forecast height error (fig. 6C) of the 500-mb. barotropic forecast was generally of the same sign (positive) as the comparable forecast for the first case (fig. 2C). Both the 500-mb. barotropic and 500-mb. thermotropic 12-hour forecast height change fields (figs. 6B and 6F) give the appearance of having been strongly smoothed when compared to the observed height change field (fig. 6D).

The 700-mb. and 500-mb. analyses were more similar in appearance at the initial time (figs. 7A and 5C) than they were 12 hours later (figs. 7C and 5D). The major difference in observed 12-hour height change at 700 mb. and 500 mb. (figs. 7D and 6D), associated with the cyclone, was over southern Illinois where a 100-ft. fall at 700 mb. and a 330-ft. fall at 500 mb. were observed. The observed 12-hour height fall centers at these two pressure surfaces in the region of New York State were in normal agreement as to relative position and magnitude. The 700-mb. 12-hour forecast height change (fig. 7B) including all four terms (figs. 8C–F) verified well in the region of the more northerly height fall center (fig. 7D). An additional height fall center was forecast over Tennessee but did not verify. However, this latter forecast height fall center at 700 mb. was in agreement with the observed 12-hour height change at 500 mb. (fig. 6D) which shows a double fall center. The 700-mb. barotropic forecast (fig. 7E) resulted in a height change forecast (fig. 7F) comparable to that for the horizontal advection term (fig. 8C). Again the 700-mb. forecast obtained by combining all four terms was superior to any of the other 700-mb. forecasts.

5. CONCLUSIONS

It is quite apparent that the horizontal advection term is of major importance at both 500 mb. and 700 mb. Of nearly equal importance at 700 mb. is the divergence term (figs. 4E and 8E). The difference in intensity of flow at the 500-mb. and the 700-mb. levels would account for the horizontal advection term being larger in magnitude at 500 mb. than at 700 mb. And also, since the fields of positive and negative values for this term are in phase at both levels, the difference in magnitude of contribution to the forecasts (figs. 2B, 4C, 6B, and 8C) can be accounted for. The 700-mb. barotropic forecast height changes (figs. 3F and 7F) compare well with the respective 700-mb. forecast height changes for the horizontal advection term (figs. 4C and 8C). When forecasts for the horizontal advection term are compared with the observed height changes at the respective pressure levels, we note that the forecast error in the magnitude of the fall center ahead of the cyclone is approximately the same percent of the observed change at both pressure levels.

From this, and also assuming that the four terms are accurately expressed and account completely for all changes in the atmosphere, we can conclude that ahead of the cyclone the contribution of the divergence term at 500 mb. can be as great in magniture and of the same sign as at 700 mb. We can then say that for these two cases the level of maximum vertical velocity ahead of the cyclone was above the assumed equivalent barotropic level of 500 mb. But we can not conclude that downstream from a cyclone or vorticity maximum, 500-mb. barotropic forecasts in all cases would produce a comparable error or even an error of the same sign. We could guess that the level of maximum vertical flow is highest in the region of upward motion immediately downstream from a cyclone, and in comparison to other synoptic regions is therefore more likely to be above a selected equivalent barotropic level in this region. Further, this is a region where the divergence term is most likely positive—a region of convergence at a selected equivalent barotropic level of 500 mb. or lower in the atmosphere when the level of maximum vertical flow is above 500 mb.—and would contribute to a forecast height fall if considered. In the region immediately upstream from a trough or cyclone where vertical motion in the troposphere is in general downward, the divergence term in equation (1) is negative in value below the level of minimum vertical velocity and positive above it.

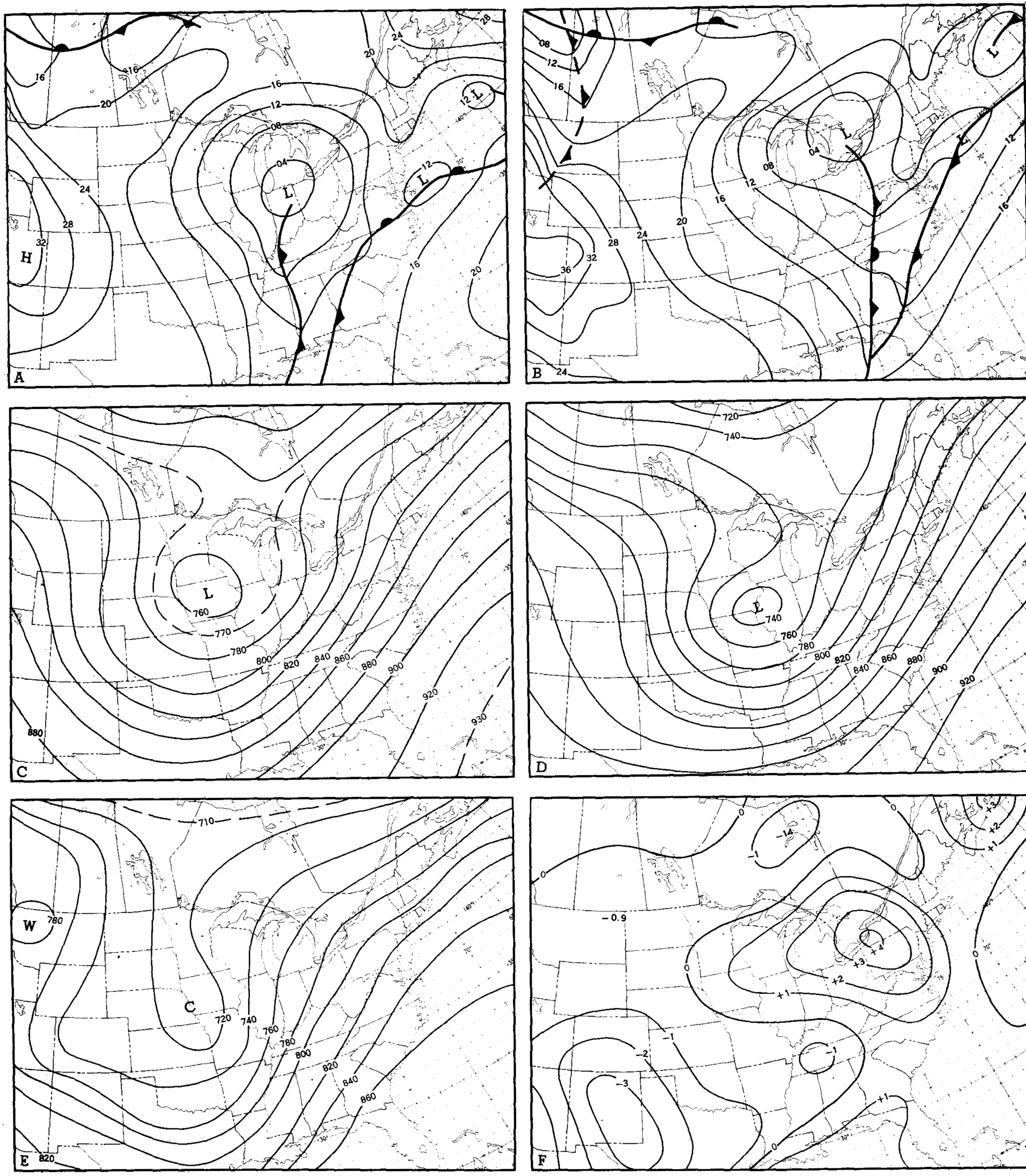

FIGURE 5.—(A) Surface analysis, 0030 and (B) 1230 GMT, Dec. 24, 1956. (C) 500-mb. analysis, 0300 and (D) 1500 GMT, Dec. 24, 1956. (E) 1000 to 500-mb. thickness analysis, 0300 GMT, Dec. 24, 1956. (F) 500-mb. vertical velocity in cm. sec.$^{-1}$, 0300 GMT, Dec. 24, 1956.

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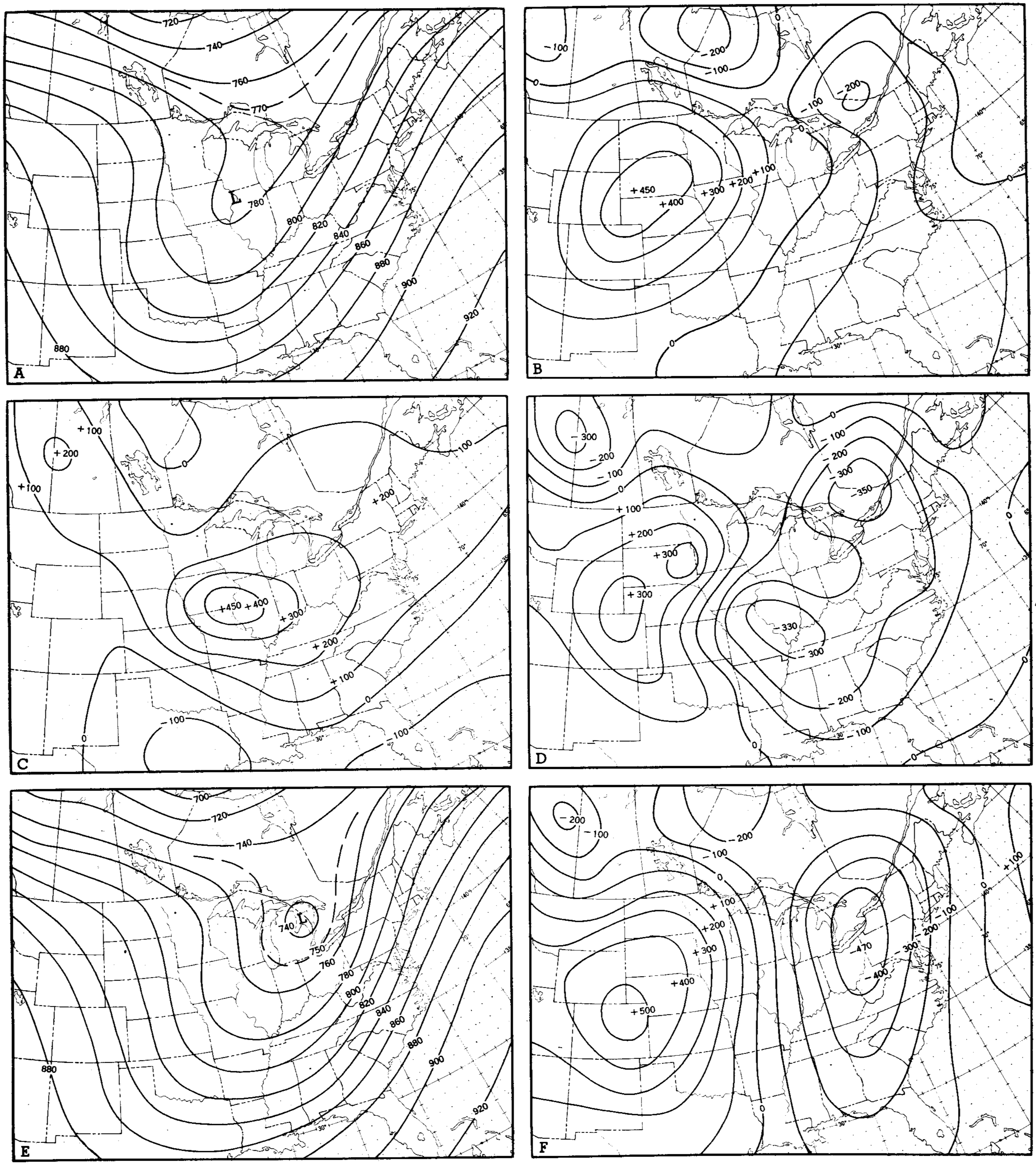

FIGURE 6.—(A) 500-mb. barotropic 12-hour forecast from 0300 GMT, Dec. 24, 1956 and (B) the height change (in feet) it represents. (C) Error of forecast height change. (D) Observed 12-hour height change from 0300 GMT. (E) 500-mb. thermotropic 12-hour forecast from 0300 GMT, and (F) the height change it represents.

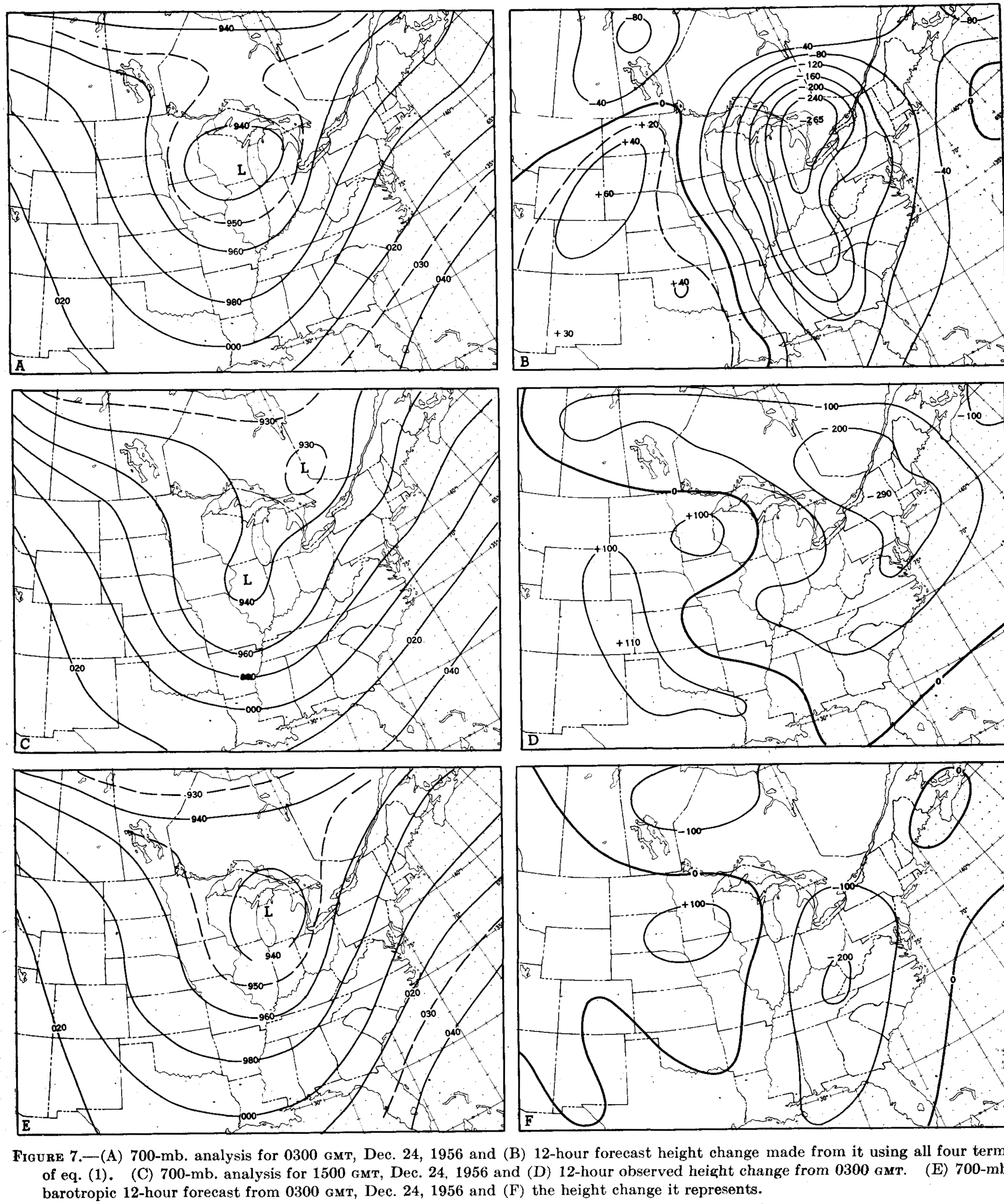

Figure 7.—(A) 700-mb. analysis for 0300 GMT, Dec. 24, 1956 and (B) 12-hour forecast height change made from it using all four terms of eq. (1). (C) 700-mb. analysis for 1500 GMT, Dec. 24, 1956 and (D) 12-hour observed height change from 0300 GMT. (E) 700-mb barotropic 12-hour forecast from 0300 GMT, Dec. 24, 1956 and (F) the height change it represents.

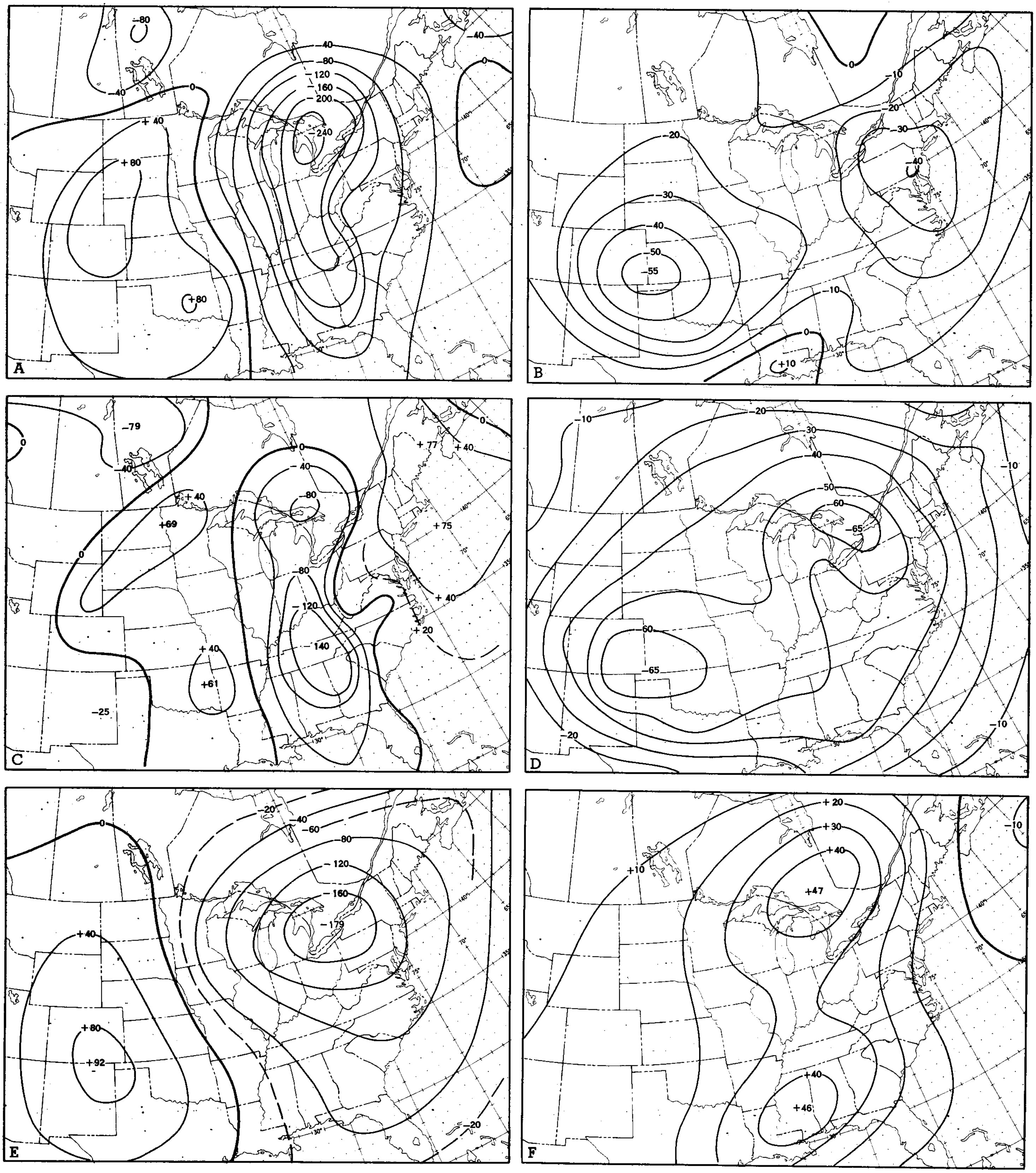

FIGURE 8.—700-mb. 12-hour forecast height change (in feet) from 0300 GMT, Dec. 24, 1956 made from (A) horizontal advection and divergence terms, $-\mathbf{V}\cdot\nabla\eta+\eta(\partial\omega/\partial p)$; (B) vertical advection and twisting terms, $-\omega(\partial\eta/\partial p)-[\nabla\omega\times(\partial\mathbf{V}/\partial p)]\cdot\mathbf{k}$; (C) horizontal advection term, $-\mathbf{V}\cdot\nabla\eta$; (D) vertical advection term, $-\omega(\partial\eta/\partial p)$; (E) divergence term, $\eta(\partial\omega/\partial p)$; (F) twisting term, $-[\nabla\omega\times(\partial\mathbf{V}/\partial p)]\cdot\mathbf{k}$.

TABLE 1.—*Mean absolute and mean algebraic values in ft./12 hours for 10×13 grid points of the individual terms of the vorticity equation at 700 mb. for 1500 GMT, October 5, 1956.*

Term	Mean absolute value	Mean algebraic value
Horizontal advection	54.5	−10.3
Divergence	21.1	−3.3
Vertical advection	6.1	3.0
Twisting	4.9	−2.5

TABLE 2.—*Mean absolute and mean algebraic values in ft./12 hours for 10×13 grid points of the individual terms of the vorticity equation at 700 mb. for 0300 GMT, December 24, 1956.*

Term	Mean absolute value	Mean algebraic value
Horizontal advection	58.4	−0.3
Divergence	18.3	1.8
Vertical advection	7.1	4.0
Twisting	7.0	−2.5

Although the magnitudes of the vertical advection and twisting terms calculated at 700 mb. for a grid point are small when compared to those of the horizontal advection or divergence terms, the values of each of these terms in equation (1) are predominantly of one sign over the entire grid, the twisting term being negative and the vertical advection term being positive and also the larger of the two terms in absolute value [5] (tables 1 and 2). Their individual contributions (figs. 4D, F and 8D, F) to a height change forecast can be a large fraction of the contributions of the horizontal advection and divergence terms (figs. 4A and 8A). To take this and the fact that they tend to counterbalance each other into consideration, a successful forecasting model would either exclude both or never include one without the other.

Although these two terms may exactly counterbalance in the mean over a large area, their fields of positive and negative contributions to the height tendency are not necessarily exactly superimposed (figs. 4B and 8B), which probably warrants their consideration in any serious attempts in extended period numerical forecasting for which the vorticity equation is employed. The twisting term should have a minimum value (greatest absolute value) at the level in the atmosphere where vertical wind shear and the horizontal gradient of vertical velocity are greatest and most nearly perpendicular [9]. This should be at or just above the level of maximum vertical flow where its magnitude should be 50 to 100 percent greater than at 700 mb. The vertical advection term, equation (1), which is a function of vertical velocity and vertical gradient of vorticity, is positive at levels in the lower troposphere where systems slope upstream with altitude; it is negative in the narrow bands between the positions of the troughs and ridges of the level for which the term is being computed and the vertically projected positions of the zero line of the 500-mb. vertical motion. When the field of the vertical advection term is relaxed to obtain a height change forecast, these narrow bands of negative values are more than counterbalanced by the predominance of surrounding positive values; however their effect can be noticeable in dividing the forecast height change field (fig. 8D) into two separate centers. The vertical advection term should have a maximum value approximately 50 percent greater than the 700-mb. value at some level near that of maximum vertical flow.

Ignoring friction, radiation, surface heating and cooling, release of heat of condensation, truncation error, error introduced by boundary assumptions, and other supposedly minor effects and considerations, we can both explain and understand the barotropic forecasting model to a certain degree. As stated, not all barotropic forecasts will verify as did the two cases presented. The level of maximum and minimum vertical flow varies in space and time. It is conceivable that the pre-selected equivalent barotropic level of the barotropic model be either above or below the level of maximum or minimum vertical flow over a large area for a period of time, which means that were the divergence term considered its contribution to the forecast at the equivalent barotropic level would be great. The relative accuracy of individual barotropic forecasts can be accounted for then by either or by a combination of two possibilities. (1) The equivalent barotropic level coincides in the mean in space and time with the level of maximum and minimum vertical flow. (2) The effects of divergence, vertical advection, and twisting terms cancel each other. If we make the reasonable assumption of no limitation in electronic computer capacity, the major problem in developing a baroclinic forecasting model as a substitute for the barotropic forecasting model is that of computing in space and time an accurate approximation of the vertical profile of vertical motion. It can be stated that the 500-mb. vertical velocity used and the vertical profile of vertical motion between 1000 mb. and 500 mb. assumed for these two case studies are subject to criticism. On the other hand independent studies [3, 5] tend to support these assumptions as do the 700-mb. 12-hour height tendency forecasts herein presented. When the problem of computing accurate vertical motion is solved finally an important milestone of progress in weather forecasting will have been passed.

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[5] 'Arnason and Carstensen [8] have since determined that inconsistent truncation in computing values of these two terms would allow the twisting term to be relatively somewhat larger in magnitude.

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